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Deviations from the Harrison-Zel'dovich spectrum due to the Quark-Gluon to Hadron Transition

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Abstract. We investigate the effect of the quark-gluon to hadron transition on the evolution of cosmological perturbations. If the phase transition is first order, the sound speed vanishes during the transition, and density perturbations fall freely. The primordial Harrison-Zel'dovich density fluctuations for scales below the Hubble radius at the transition develop peaks, which grow linearly with the wavenumber, both for the hadron-photon-lepton fluid and for cold dark matter. The large peaks in the spectrum produce cold dark matter clumps of 10^{-8} to $10^{-11} M_\odot$.

QCD makes a transition from a quark-gluon plasma at high temperatures to a hadron gas at low temperatures. Lattice QCD simulations give a transition temperature $T_\star \sim 150$ MeV and indicate a first-order phase transition for the physical values of the u,d,s-quark masses [1]. The relevance of the QCD transition for cosmology, especially for big-bang nucleosynthesis [2], has been discussed before, but the focus was on effects of bubble formation. In this paper and in [3] we look at matter averaged over scales much larger than the bubble separation. We show that for a first order phase transition the sound speed $c_s = (\partial p/\partial \rho)_s^{1/2}$ drops to zero suddenly at the moment the transition temperature $T_\star$ is reached, stays zero for the entire time until the phase transition is completed, and afterwards suddenly rises back to $c_s \approx c/\sqrt{3}$. In contrast the pressure stays positive and varies continuously, although it goes below the radiation fluid value $p/\rho = 1/3$. Since c_s is zero during the transition, there are no pressure perturbations, no pressure gradients, no restoring forces. Pre-existing cosmological perturbations, generated by inflation with a Harrison-Zel'dovich spectrum, go into free fall for about a Hubble time. The superhorizon modes (at the time of the transition) remain unaffected, the subhorizon modes develop peaks in $\delta\rho/\rho$ which grow linearly with

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wavenumber, $\sim k/k_\star$, where $k_\star^{\rm phys} \sim$ Hubble rate H at the end of the QCD transition.

The sound speed, $c_s = (\partial p/\partial\rho)_s^{1/2}$, must be zero during a first-order phase transition of a fluid with negligible chemical potential, since the fluid must obey

$$\rho + p = T\frac{dp}{dT} \ , \tag{1}$$

according to the second law of thermodynamics. Because the energy density ρ is discontinuous in temperature at $T_\star$ for a first-order phase transition, the pressure p must be continuous with a discontinuous slope. As the universe expands at fixed temperature $T_\star$ during the phase transition, ρ as a function of time slowly decreases from $\rho_+(T_\star)$ to $\rho_-(T_\star)$, p stays constant at $p(T_\star)$, and therefore c_s is zero during the whole time of the phase transition.

The interaction rates in the QCD-photon-lepton fluid are much larger than the Hubble rate, $\Gamma/H \gg 1$, therefore we are very close to thermal and chemical equilibrium, the QCD transition is very close to a reversible thermodynamic transformation. Estimates show that supercooling, hence entropy production, is negligible, $(T_\star - T_{\rm supercooling})/T_\star \sim 10^{-3}$ [4]. Bubble formation is unimportant for our analysis, estimates give a bubble separation $\ell_b \sim 1$ cm [5], while the Hubble radius at the QCD transition is $R_H \sim 10$ km. We shall analyze perturbations with $\lambda \gg \ell_b$.

The bag model gives a parameterization and a reasonable fit to the lattice QCD data [6]. In the bag model it is assumed that for $T > T_\star$ the quark-gluon plasma (QGP) obeys

$$p_{\rm QGP}(T) = p_{\rm QGP}^{\rm ideal}(T) - B \ , \tag{2}$$

where $p_{\rm QGP}^{\rm ideal}(T) = (\pi^2/90) g_{\rm QGP}^* T^4$, g^* is the effective number of relativistic helicity states, and B is the bag constant. We include u,d-quarks and gluons in the quark-gluon plasma, and for $T < T_\star$ we have a hadron gas (HG) of pions, which we treat as massless and ideal. ρ follows from Eq. (2) via the second law, Eq. (1), and s from $s = dp/dT$. The bag constant is determined by the critical temperature $T_\star$ via $p_{\rm QGP}(T_\star) = p_{\rm HG}(T_\star)$. The photon-lepton fluid contains γ, e, μ, and 3 neutrinos. The growth of the scale factor during the QCD transition, $a_+/a_- \approx 1.4$, follows from the conservation of entropy in a comoving volume.

The evolution of linear cosmological perturbations through the QCD transition is analyzed in the longitudinal sector (density perturbations) for perfect fluids. We choose a slicing Σ of space-time with unperturbed mean extrinsic curvature, $\delta[{\rm tr}K_{ij}(\Sigma)] = 0$. The adapted gauge is the uniform expansion (Hubble) gauge [7]. As fundamental evolution equations for each fluid we have $\nabla_\mu T^{\mu\nu} = 0$, i.e. the continuity equation and (in the longitudinal sector) the 3-divergence of the Euler equation of general relativity,

$$\partial_t \epsilon = -3H(\epsilon + \pi) - \triangle\psi - 3H(\rho + p)\alpha \tag{3}$$

$$\partial_t \psi = -3H\psi - \pi - (\rho + p)\alpha \ , \tag{4}$$

where $\epsilon \equiv \delta\rho$, $\pi \equiv \delta p$, $\rho \equiv \rho_0$, $p \equiv p_0$, $\vec\nabla\psi =$ momentum density, $\alpha =$ lapse function. The system of dynamical equations is closed by Einstein's $R_{\hat 0\hat 0}$-equation, the general relativistic Poisson equation,

$$(\triangle + 3\dot H)\alpha = 4\pi G(\epsilon + 3\pi) \ , \tag{5}$$

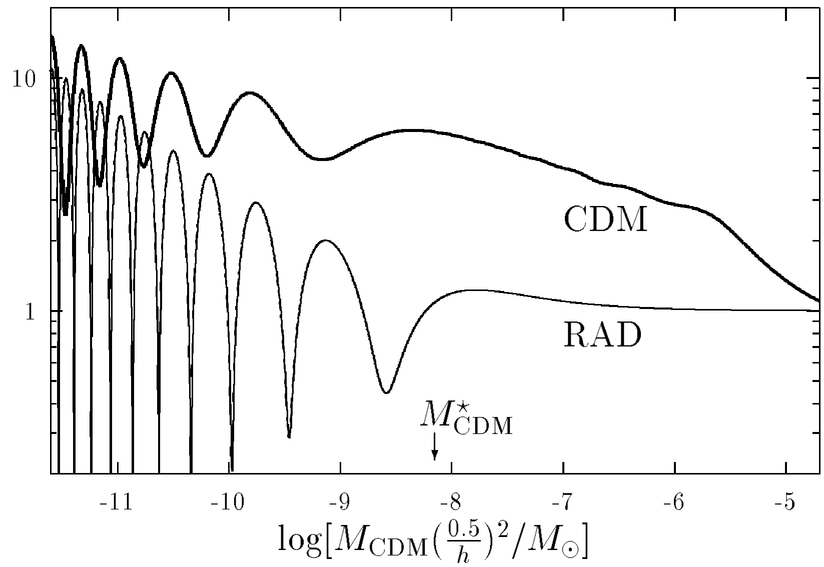

Figure 1: The modifications of the CDM density contrast $A_{\mathrm{CDM}} \equiv |\delta_{\mathrm{CDM}}|$ at $T_\star/10$ and of the radiation fluid amplitude $A_{\mathrm{RAD}} \equiv (\delta^2_{\mathrm{RAD}} + 3\hat{\psi}^2_{\mathrm{RAD}})^{1/2}$ due to the QCD transition. Both quantities are normalized to the pure Harrison-Zel'dovich radiation amplitude. On the horizontal axis the wavenumber k is represented by the CDM mass contained in a sphere of radius π/k.

together with the equation of state. These three equations are the Jeans equations extended to general relativity in the longitudinal sector.

Our numerical results of evolving a mode k of a cosmological perturbation through the QCD transition are given in Fig. 1. We work with the dimensionless variables $\delta \equiv \epsilon/\rho$ (density contrast) and $\hat{\psi} \equiv k^{\mathrm{phys}}\psi/\rho$ ($\sim$ peculiar velocity). We show the enhancement of the amplitude $A_{\mathrm{RAD}} \equiv (\delta^2_{\mathrm{RAD}}+3\hat{\psi}^2_{\mathrm{RAD}})^{1/2}$ of the acoustic oscillations of the radiation fluid (QCD, photons, leptons) after the transition compared to the amplitude without transition. For cold dark matter (CDM) we show the amplitude $A_{\mathrm{CDM}} \equiv |\delta_{\mathrm{CDM}}|$ at $T_\star/10$ compared to A_{RAD} without transition. In both cases we obtain peaks over the Harrison-Zel'dovich spectrum of adiabatic density fluctuations. Only those modes are affected which are subhorizon at the end of the transition, $k \gtrsim k_\star$, where $k^{\mathrm{phys}}_\star(t_+) \sim H(t_+)$. Our peaks grow linearly in k for $k \gg k_\star$. The modes k are labeled by the CDM mass contained in a sphere of radius $\lambda/2 = \pi/k$. The value $k_\star$ corresponds to $M^\star_{\mathrm{CDM}} \sim 10^{-8} M_\odot$. The radiation energy inside $\lambda_\star/2$ is $\sim 1 M_\odot$, but it gets redshifted as $M_{\mathrm{RAD}}(a) \sim (a_{\mathrm{equality}}/a) M_{\mathrm{CDM}}$. The perturbations in the radiation fluid will be wiped out by collisional damping from neutrinos on scales much smaller than the horizon scale at neutrino decoupling (1 MeV).

For cold dark matter we consider the lightest supersymmetric particle (LSP), the neutralino $\tilde{\chi}^0_1$ in the minimal supersymmetric standard model with universal supersymmetry breaking at the grand unified scale. LEP 1.5 combined with the gluino mass limit from Fermilab gives a minimal mass $M^{\mathrm{min}}_{\mathrm{LSP}} = 27$ GeV [8]. With a freeze-out temperature of $T_f \sim M_{\mathrm{LSP}}/20$ free-streaming wipes out CDM structure for $k/k_\star > 10$.

The origin and magnitude of these impressive peaks for $k \gg k_\star$ is easily understood. The radiation fluid in each subhorizon mode makes standing acoustic oscillations before and after the QCD transition with gravity negligible and with equal amplitudes of δ and $\sqrt{3}\hat{\psi}$. During the transition the sound speed is zero, there are no restoring forces from pressure gradients, the radiation fluid goes into free fall. If the transition time is short, $\Delta t < H^{-1}$, gravity is again negligible for the radiation fluid during this free fall. This is inertial motion in the sense of Newton. The peculiar velocity is constant, and the density contrast grows linearly in time with a slope k. Therefore the height of the peaks is $(A_+/A_-)_{\rm peaks} \approx k/k_\star$. Hence modes with $k^{\rm phys}/H \gtrsim 10^4$ go nonlinear by the end of the QCD transition. The explicit solution along these lines is presented in [3]. CDM falls into the gravity wells generated by the radiation fluid.

The peaks above the Harrison-Zel'dovich spectrum lead to CDM clumps with $10^{-8} > M_{\rm CDM}/M_\odot > 10^{-11}$, which go nonlinear sometime after equality and virialize by violent gravitational relaxation. The mass range of these CDM clumps lies just beyond the smallest mass ($5 \times 10^{-8} M_\odot$) accessible to present microlensing observations [9], thus future lensing observations might discover such clumps.

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